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Reflections on Student Performance in Area and Volume in the Punjab

Edward Redden*, Annes Fatima** & Muhammad Bakhtiar***

Abstract

This paper investigates the interaction between curriculum design and student performance in two subtopics of the mathematics curriculum in the Punjab, Pakistan. A conceptual framework is presented that describes a structure for concept development in measurement and this is compared with the results of a content analysis of Punjab curriculum documents. Serious shortcomings in the curriculum are identified and it is argued these shortcomings are reflected in students' achievement of outcomes that are explored in the grade 5 and 8 examinations conducted by the Punjab Examination Commission.

Introduction

The Ministry of Education, Punjab has begun a project to monitor student performance in all subjects at the end of grade 5 and the end of grade 8. Traditionally, such an examination process has been undertaken for the sole purpose of making judgments about the students' readiness to progress to the next grade. As a consequence the focus was on producing a norm referenced mark for each student and to make an arbitrary decision as to what constituted a 'pass'. Additionally, each of the 36 districts of the Punjab produced their own examination papers thus making it impossible to provide advice with regard to variability in standards across the Province as a whole.

In 2006 a decision was made to centralize the examination process and to modify the design of the process to allow advice to be provided for a wide range of educational processes. These are summarized on the PEC website (PEC 2009) as:

> The Examinations arranged under PEC provide highly accurate results that can be used to:
>
> - Identify the skills and concepts individual students have learnt
> - Diagnose instructional needs.
> - Monitor academic growth over that time.

*Senior Education Specialist. Australia
**Secondary School Teacher. Chakwal
***Senior Subject Specialist. Lahore

- Make data-driven decisions at school, Tehsil and district levels.
- Determine the future/destination of the education system.

From this it can be inferred that there is a shift from merely reporting pass/fail decisions to providing data to support advice on the quality of instructional processes, resource allocation and variability of the quality of the education process across the Punjab. This has been facilitated by entering data into a data base at item level, thus facilitating a form of analysis known as item response theory that has been operationalised as a one parameter model using Rasch Analysis (Bond and Fox, 2007). This data has now been entered and saved over a four year period for approximately 1.2 million grade 5 students and 0.8 million grade 8 students, thus providing a very rich data source for educational researchers.

The authors of this paper were recently preparing an *Advice for Teachers Report* (Redden, Fatima and Bakhtiar 2009) based on the item response data from the 2009 examination round and began to reflect on the performance of students on questions relating to area and volume as sub-topics of the measurement strand of the mathematics curriculum. It is very rare for researchers to have access to data from a population of 2 million students over a period of 4 years.

Much assessment of student performance in the popular press places 'blame' for poor performance on teacher quality. The design of the Punjab examination process makes explicit the use of a systems approach (Windham 1988) to such analysis in which a range of 'inputs' are mixed in a 'process' that results in 'outputs' which in this context are student responses to examination questions. The process is best conceived of as the classroom and school activity and the inputs are the various resources that contribute to the classroom and school activity.

The inputs are many and varied and, it needs to be emphasized, include teacher quality as focused on in the popular press. But it also includes many other things including family background of students, physical school resources and the curriculum that defines what is to be taught. The curriculum here is defined in its broadest sense and includes those elements that define classroom activity. These might be considered as consisting of the mathematics syllabus, the mathematics text books, teacher manuals, and other supplementary teaching resources.

In this paper we focus our attention on the curriculum and text books as the inputs of interest and explore their quality against a conceptual framework for the development of measurement concepts in the international literature.

The deliberations of the authors led to the formulation of two research questions:

- How well have the students of Punjab in grade 5 and grade 8 performed on questions exploring their understanding of area and

volume from the mathematics curriculum?
- Does the Punjab curriculum reflect international best practice for the measurement concepts of area and volume?

To address these question this paper is organized into four sections following this introduction. The first is a literature review that presents the conceptual framework. This is followed by a description of the methodology used to explore the research questions. The third section discusses the data for the concepts of area and volume separately and finally, a set of conclusions is drawn from the discussion.

Literature Review

For nearly 20 years there has been a standard approach (Board of Studies 1989, NCTM Standards, 1989) to the developing of concepts that involve measurement. This approach can be applied to the very first concept of measurement which we would generally accept to be the measurement of length, through to much more sophisticated constructs first met in senior high school such as momentum or components of electricity. This approach avoids the pitfall of assuming the construct of interest already exists in children's minds and all teachers need to do is present the formal abstractions of the construct and the necessary skills will be retained.

To exemplify this approach the experience of a colleague is recalled. The colleague's son came home from school and proudly announced that his class "had done volume today". The father, always ready to encourage such learning asked what is the volume of a box 4 cm by 3 cm by 5 cm. The son, being a bright lad quickly offered 60 as the answer. The father ignored the issue of suitable units and asked a few more similar questions all of which were correctly responded to. He then gave his son a cardboard carton and a ruler and asked "what is the volume of the box." The son looked at him completely perplexed and it was clear he did not know what the father's question was about. It soon became very obvious that the son had learned to apply an algorithm to three numbers without any understanding of what the attribute of volume was all about.

The conceptual framework for measurement activities comes in a variety of forms but is generally based on the following sub-concepts:

- Aware of attribute
- Ordering of two objects of different size
- Order a set of objects of varying sizes
- Informal units to measure the concept such as hand prints for area or blocks of a similar size for volume
- Need for formal units due the variability of the informal units
- Formal unit defined and named
- Need for a range of formal units and being able to choose an

appropriate unit

- Formal processes for deriving measurements indirectly such as the use of formula.

In the experience of our colleague described above it is clear that the experiences given to the student were restricted to the last of these activities without due attention being paid to the prerequisite activities outlined in which an effort is made to build a clear understanding of the underlying concepts of the attribute to be measured. These difficulties were known to exist by Piaget (1967) and underlie many of his famous conservation tasks (Piagetian Conservation Task 2007)

The National Council for Teachers of Mathematics (2009a) has produced a generic measurement outline to cover the measurement concepts for length, volume, weight, area, and time. These cover experiences for grades 1 to 12 and while they do not map exactly onto the framework outlined here, careful consideration allows a mapping of between the two models

In the state of Victoria, Australia, a much simpler model is outlined on the website (Victorian Curriculum 2009) that identifies three phases each with several steps that they say is required for effective teaching of measurement concepts.

Phase 1 - Identifying the attribute

- Develop the concept of the attribute.
- Distinguish it from other attributes.
- Gain intuitive understanding of properties.

Phase 2 - Learning to measure

- Learn to measure the quantity.
- Use formal units.
- Estimate measurements.

Phase 3 - Learning to calculate

- Convert from one unit to another.
- Calculate, instead of direct measure.

Both of these examples can be seen to map with considerable agreement to our conceptual framework outlined at the beginning of this section. It is not claimed here that the framework being offered is the ‘only’ framework, but rather that there is general agreement in international literature about the necessary approaches to the teaching of measurement concepts.

In our outline of the systems approach above it was indicated that the attention here was to focus on the curriculum issues surrounding measurement of area and volume and to avoid the issue of teacher quality. It is clear that the two cannot always be separated. If the curriculum does not represent best practice, it cannot be expected that the teachers can be expected to reflect best practice in the classroom. Thus it can be argued the tendency of the popular press to focus on teacher quality without seeing the interaction of many inputs into the system is a very simplistic approach and unfairly allocates the blame for shortcomings in the education system.

Method

To explore these issues further two sources of data have been used, each associated with one of the research question presented above.

The first is a content analysis of current curriculum documents. These include the syllabus underlying the current pattern of instruction in Punjab, the new Pakistan mathematics curriculum (2006) and the text books that are currently available. It should be pointed out that only the text books are commonly available for teachers as syllabus documents are not distributed to schools. Further the text books are based on the old syllabus as they have not as yet been updated to reflect the approach of the 2006 syllabus documents. However, the content of the 2006 documents are analyzed here to provide an indication of new directions. The content of these documents will be compared with our conceptual framework outlined in the previous section.

The student achievement data will be extracted from the PEC database. This data has been entered at item level so the responses of 2 million children for each of the years 2006 to 2009 are accessible. Further this data has been subjected to Rasch analysis using Quest software (Adams and Khoo 1994) and thus psychometric properties of each item are available including item difficulty, item fit, and discrimination index. In addition, the software produces a variable map that facilitates the mapping of item difficulty against student performance. Most of the test items being considered are of multiple choice types and have been marked in a binary code of right or wrong. These are relatively simple to interpret. Some of the data is in the form of open-ended questions that have 3 parts and a range of marks allocated from 0 to 5. These are more complex to analyze. However, in designing the open-ended questions the SOLO Taxonomy (Biggs and Collis 1982) was used as a conceptual framework to assist in making judgments with regard to the quality of students responses. The SOLO Taxonomy provides a series of sequenced levels of performance as observed in student responses. In the design of the open-ended questions the test items were designed to require a response at three of the levels. The three levels were *uni-structural*, which require a single operation, *multi-structural* which requires a series of operation in a previously learnt sequence and *relational* which requires an overview and integration of all the data elements. Responses that earn no marks are called *pre-structural* responses. This model has been used to assist in interpreting the quality of student responses.

Results

Area

In Table 1 the content analysis of the old and new curriculum documents for the concept of Area is summarized. The old curriculum documents consist only of the text books produced by the Punjab Text Book Board as there are no syllabus documents available. The new curriculum is the syllabus document produced in 2006 as the National Curriculum in Mathematics. The support materials for this new syllabus have not yet been developed.

Table 1
Area content analysis Old and New Curriculum

Grade	1		2		3		4		5		6		7		8	
Area	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New
Awareness									X	X						
Comparison of 2 regions																
Ordering of a set of regions																
Use of informal units																
Need for formal units									X							
Formal units									X	X	X	X	X	X	X	X
Use of most appropriate units									X							
Formal processes for deriving measurements indirectly such as the use of formula.									X	X	X	X	X	X	X	X

Observations for Content Analysis: Area

In Grade 5 the concept of area is introduced directly. This is achieved by giving the formula i.e. for the “area of square you multiply side by side” and for “area of a rectangle you multiply length by width”. No attempt is made to develop an informal or intuitive understanding of the notion of area nor are any practical exercises recommended to assist students understanding in the table 1.

During the content analysis process for Area and Volume it was noted that the measurement content of Length was more complete. In Grade 1 informal units to measure length (like match sticks to measure dimensions of books and hands to measure length of tables) are introduced. In Grade 2 formal units to measure length are introduced. Students learn how to use a measuring tape to measure the length of a room, table etc. Multiple and sub multiples of units of length like m and cm. Also addition and subtraction of two measurements is introduced. In Grade 3, the conversion between meters and centimeters, and kilometers to meters is treated but no equivalent discussion in the area construct is undertaken. In Grade 4, mathematical operations, conversion of units is repeated but, again, there is no equivalent discussion in relation to units of area.

Thus, it can be seen that the theoretical constructs in the conceptual framework being used here seem to be known to the curriculum writers and text book developers but are only being applied to the very simplest of measurement constructs. However, they are not being applied to the more advanced construct of Area in the lower grades. At times opportunities for an appropriate treatment of informal area concepts are overlooked. For example in Grade 2 the introduction of open and closed figures is discussed. However the discussion is restricted to the treatment of closed figures i.e. boundary and interior and does extend to the concept of area.

In Grade 6, 7 and 8, the emphasis is on the use of formula for determining area of plane figures. These include triangle parallelogram and trapezium (Grade 6), surface area of a cylinder (Grade 7), surface area of cone and sphere (Grade 8). The use of Hero’s formula is introduced in Grade 8. Some questions from daily life are also in these exercises, including complex diagrams incorporating multiple plane figures.

It would seem from this discussion that little emphasis is paid in the curriculum to building the foundations for an understanding of the Area concept upon which the more formal operations can be built in grades 6 to 8. This has implications for the quality of student’s responses to area questions. These quality issues are now discussed using date from the Punjab Examination Commission’s Grade 5 and Grade 8 examination papers for 2009.

Table 2 provides the frequency of correct responses to questions exploring children’s understanding of student learning outcomes (SLO) associated with the area concept in the 2008 and 2009 round of examinations for grade 5 and grade 8.

The data available data from grade 5 is restricted to two multiple choice questions. The easiest of which was a simple application involving a 6 by 4 rectangle and only 64% of students provided a correct choice of answers. 12% calculated the perimeter, 7% squared the 4 and 11 % squared the 6. The harder question involved deriving the rectangle dimensions from a diagram and then finding the area of a 4 by 2 rectangle. Only 35% of students successfully identified the correct answer. While the frequency of correct responses to these MCQs is alarming the situation was considerably worse in the open ended questions.

Table 2
Frequency of Correct Responses on Area SLO's (2008-09)

Item No.	SLO	% correct Response 2009	% correct Response 2008
MCQ's Grade 5			
6	Find area of simple rectangular diagram	64	
22	Find area of shaded rectangle from a complex diagram	35	
MCQ's Grade 8			
28	Find area of triangle using Hero's formula	28	
23	Find curved surface area of a right circular cone	30	
35	Find length of a square if area is given.	50	
34	Find surface area of sphere if radius is given		38
40	Apply concept of area of curved surface to a complex daily life problem		12
22	Find surface area of the right circular cone if radius and height is given	42	37

For the SLO of "Find area of complex diagram" Students were asked to "find area of the un-shaded Region" of Figure 1 if the side of each square is 1 cm long. Only 10% provided the correct answer and 43% of students made no attempt

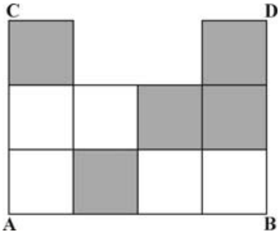

Figure 1

For the SLO of "Apply the concept of area to daily life problems" the following question was asked:

> Length of single brick wide wall is 5m and its height is 2m. If length of one brick is 25 cm and height is 20 cm then calculate number of bricks required to construct this wall.

Only 7% provided the correct response.

It is clear from this data that very few of the grade 5 students are displaying behavior that reflects achievement of the curriculum outcomes.

The data for grade 8 provides a similar pattern. From table 2 it can be seen that the frequency of correct responses for area questions range from

50% for finding the length of the side of a square given its area to 12% for "Apply concept of area of curved surface to a complex daily life problem" with the changes in frequency being consistent with the increase in complexity of the SLO.

As with Grade 5, the Grade 8 performance on open ended items was considerably worse.

84% were able to "find the area of given rectangle" when the rectangle was a simple one as in figure 2

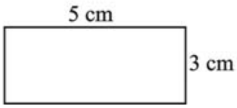

Figure 2

46% were able to "find area of given triangle using Hero's Rule when given the formula to apply (figure 3)

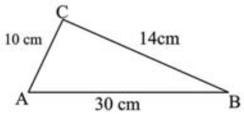

Figure 3

Finally only 9% were able to "find the area of quadrilateral ABCD" in figure 4.

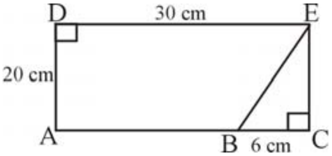

Figure 4

Volume

Table 3 provides an overview of the curriculum content for the concept of Volume. This table provides some references to awareness. However, closer inspection reveals that these experiences are related to capacity (how much a container holds) rather than volume (the amount of space occupied by something). While this technical distinction exists, the early capacity experience could be expected to provide some early concept building. The work indicated for grades 1 to 4 in the old curriculum involves the use of milliliters and liters and the conversion of one unit to another. Unfortunately the curriculum materials reviewed do not make the link between capacity and volume, nor do they provide the necessary preparation for the formal treatment of volume. Hence volume is introduced in grade 5 with the direct application of formula without any underlying concept building using informal strategies.

Table 3
Frequency of Correct Responses on Volume SLO’s (2008-09)

Grade	1		2		3		4		5		6		7		8	
Area	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New
Awareness	X		X													
Comparison of 2 regions	X			X	X											
Ordering of a set of regions																
Use of informal units	X															
Need for formal units																
Formal units			X	X	X	X	X	X	X	X	X	X	X	X	X	X
Use of most appropriate units				X												
Formal processes for deriving measurements indirectly such as the use of formula				X	X	X	X	X	X	X	X	X	X	X	X	X

The set of applications grows in complexity with the increase in grade. Cubes and rectangular prisms are treated in grades 5 and 6, cylinder in grade 7 and cone and spheres in grade 8.

The resulting competencies of students from this lack of focus on concept development is evident in the item facilities reported in Table 4. This table provides the percentage of correct responses to the MCQs on volume from the years 2008 and 2009. These range from 65 % getting the very simplest of questions correct to only 11 % getting the more complex questions correct. This data cannot convince us that a large proportion of the candidature is successfully demonstrating the outcomes required of the syllabus.

Table 4
Frequency of Correct Responses on Volume SLO’s (2008-09)

item No.	SLO	% correct Response 2009	% correct Response 2008
MCQ’s Grade 5			
29	Find volume of cuboid shape from the daily life problems	55	
6	Find volume of a cuboids’ if area of base and height is given		65
MCQ’s Grade 8			
32,39	Find volume of cone if area of base and height is given	26	35
40	Apply concept of area of curved surface to a complex daily life problem		12
22	Find area of the right circular cone if radius and height is given	42	37
29	Find volume of sphere if radius of sphere is given	11	

Unfortunately there were no open ended items on the topic of volume in 2008 or 2009

Conclusion/Recommendations

A number of observations can be made from the data reported above. The first is that both the old and the new curriculum are not consistent with the conceptual framework that was derived from international literature. This has resulted in a lack of conceptual understanding of the underlying constructs and a resultant reliance on rote learning of formula and procedures that can be applied without sophisticated understanding of issues. This perception can be better understood by further exploration of the opened item included in the above analysis. The graph below (figure 5) reports an analysis of the grade 8 area item based on a classification of the responses using the SOLO Taxonomy (Biggs and Collis 1982).

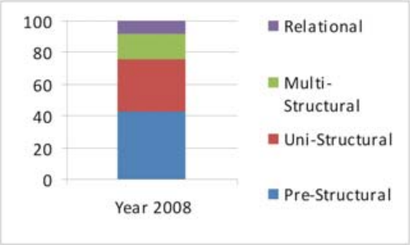

Figure 5

This hierarchical model of cognitive growth uses classifications ranging from no understanding evident (Pre-structural) to a deep and sound understanding in which a student can demonstrate an overview of all the data in the question and the ability to integrate that data (Relational). This relational understanding should be the aim of classroom instruction. The graph below indicates the distribution of students across the categories of the taxonomy and points to the very few students achieving the desired quality of outcome.

The conceptual framework would indicate that the use of informal units, comparison of objects having the attribute under investigation and the need for standardized units should be introduced. In addition there should be a continuity of concept growth from grade 1 to grade 8.

This should be supported with investigation activities using concrete examples incorporated into the curriculum experience of children. These would probably be best placed in teacher manuals which detail classroom activity and encourage the development of lessons involving conjecture, hypothesizing and the testing of ideas that will assist in building a conceptual understanding prior to the formal introduction of methods and procedures.

Such techniques will need to be carefully developed in in-service and pre-service teacher education programs. Such approaches are currently embedded in the National Professional Standards for Teachers that will be used to guide the future teacher education curriculum. However, these standards cannot be implemented unless there is adequate support for teachers in the form of teaching resources and appropriate curriculum.

Another issue that becomes apparent from this analysis is the great variability in achievement among the students in both grade 5 and grade 8. It needs to be investigated if the current curriculum is appropriate for all students in these grades. Are the concepts being directed at the appropriate age groups? It seems clear that the current curriculum needs some flexibility to adjust the classroom experiences of students based on their previous achievement and innate capacity in the concept being studied. The catering for individual differences is one of the great challenges of classroom teaching, especially in a context of large class sizes, poorly trained teachers and inadequate resources.

Early in this paper, it was argued that the 'blaming of teachers' for student performance was not consistent with the complexity of the educative process. Here we have pointed to the necessary interaction of curriculum, teaching resources, and teacher training for the successful achieving of desired outcomes especially in the concepts of area and volume in elementary school.

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